Basavarajeswari Group of Institutions

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | |

Course Code | 2 | 2 | M | D | A | 3 | 1 |

Third Semester B.E. Degree Examinations, January 2025

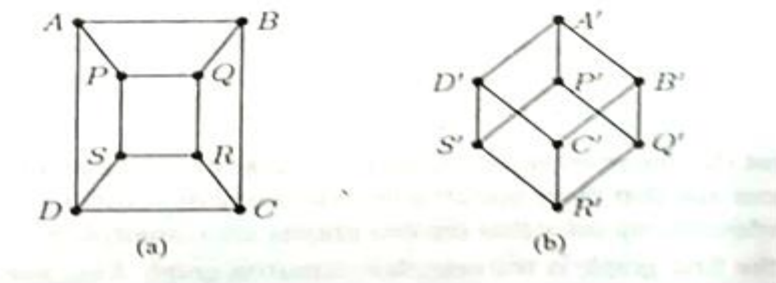
Graph Theory and Discrete Mathematical Structures, Probability and Statistics

(Common to AIML, CSE-AI and CSE-DS)

Duration: 3 hrs **Max. Marks: 100**

Note: *1. Answer any* **FIVE** *full questions choosing ONE Full Question from each Module*
2. Formula Handbook is permitted
3. Missing data, if any, may be suitably assumed

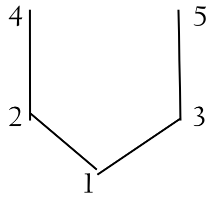
Q. No		***Question***	***Marks***	***(RBTL:CO:PI)***
		Module-1		
1.	**a.**	Define with examples (i) Complete graph (ii) Regular graph and (iii) Bipartite graph	**06**	(2 :1 : 1.2.1)
	b.	If $G = G(V, E)$ is a simple graph, prove that $2\|E\| \le \|V\|^2 - \|V\|$.	**07**	(2 :1 : 1.2.1)
	c.	Prove that in every graph, the number of vertices of odd degrees is even.	**07**	(2 :1 : 1.2.1)
		(OR)		
2.	**a.**	Verify that the two graphs shown below are isomorphic	**06**	(2 :1 : 1.2.1)



	b.	In the complete graph with n vertices, where n is an odd number ≥ 3, there are $(n-1)/2$ edge-disjoint Hamiltonian cycles.	**07**	(2 :1 : 1.2.1)
	c.	Draw the following (i) A graph which has both an Euler circuit and Hamilton cycle. (ii) A graph which has an Euler circuit but no Hamilton cycle. (iii)A graph which has a Hamilton cycle but no Euler circuit. (iv)A graph which has neither a Hamilton cycle nor an Euler circuit.	**07**	(2 :1 : 1.2.1)
		Module-2		
3.	**a.**	Let $A = \{1, 2, 3, 4\}$ and let R be the relation on A defined by xRy if y=2x. (i) Write down R as set of ordered pairs. (ii) Draw the digraph of R. (iii) Determine the in-degree and out-degree of the vertices in the digraph.	**06**	(2 :2 : 1.2.1)
	b.	For a fixed integer $n > 1$, prove that the relation "congruent modulo n" is an equivalence relation on the set of all integers z.	**07**	(2 :2 : 1.2.1)

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

c. Determine the matrix of the partial order whose Hasse diagram is given below: **07** (2 :2 : 1.2.1)



(OR)

4. a. Let f and g be functions from R to R defined by $f(x)=ax+b$ and $g(x)=1-x+x^2$. If $(gof)(x)=9x^2-9x+3$, determine a,b. **06** (2 :2 : 1.2.1)

b. Let $p=\begin{pmatrix}1 & 2 & 3 & 4 & 5 & 6\\ 4 & 3 & 5 & 1 & 2 & 6\end{pmatrix}$ be a permutation of the set $A=\{1,2,3,4,5,6\}$ **07** (2 :2 : 1.2.1)

(i) Write p as a product of disjoint cycles (ii) Compute p^{-1} (iii) Compute p^2 and p^3.

c. Let $f:A\to B, g:B\to C$ and $h:C\to D$ be three functions. Prove that $(hog)of=ho(gof)$. **07** (2 :2 : 1.2.1)

Module-3

5. a. Solve the recurrence relation $a_n=7a_{n-1}$, where $n\ge 1$, given that $a_2=98$. **06** (2 :3 : 1.2.1)

b. Solve the recurrence relation $2a_n-3a_{n-1}=0$, for $n\ge 1$, given that $a_4=81$. **07** (2 :3 : 1.2.1)

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **07** (2 :3 : 1.2.1)

(OR)

6. a. Solve the recurrence relation $F_{n+2}=F_{n+1}+F_n$ for $n\ge 0$, given $F_0=0, F_1=1$. **06** (2 :3 : 1.2.1)

b. Solve the recurrence relation $a_{n+2}+4a_{n+1}+4a_n=7$, $n\ge 0$. given that $a_0=1$, $a_1=2$. **07** (2 :3 : 1.2.1)

c. Solve the recurrence relation $a_{n+2}-10a_{n+1}+21a_n=3n^2$, $n\ge 0$. **07** (2 :3 : 1.2.1)

Module-4

7. a. The following data gives the age of husband (x) and the age of wife (y) in years. Form the two regression lines and calculate the age of husband corresponding to 16 years age of wife. **06** (2 :4 : 1.2.1)

x	36	23	27	28	28	29	30	31	33	35
y	29	18	20	22	27	21	29	27	29	28

b. If θ is the acute angle between the lines of regression, then show that **07** (2 :4 : 1.2.1)

$$\tan\theta=\frac{\sigma_x\sigma_y}{\sigma_x^{\,2}+\sigma_y^{\,2}}\left(\frac{1-r^2}{r}\right).$$

Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)

c. Ten students got the following percentage of marks in two subjects x and y. Compute their rank correlation coefficient **07** (2 :4 : 1.2.1)

Marks in x	78	36	98	25	75	82	90	62	65	39
Marks in y	84	51	91	60	68	62	86	58	53	47

(OR)

8. **a.** Find the equation of the best fitting straight line for the following data and hence estimate the value of the dependent variable corresponding to the value 30 of the independent variable. **06** (2 :4 : 1.2.1)

x	5	10	15	20	25
y	16	19	23	23	30

b. Fit a parabola $y = ax^2 + bx + c$ for the following data **07** (2 :4 : 1.2.1)

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

c. Fit a least square geometric curve $y = ax^b$ for the following data. **07** (2 :4 : 1.2.1)

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

Module-5

9. **a.** A random variable X has the following probability functions for the various variables of x. **06** (2 :5 : 1.2.1)

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k (ii) Evaluate $P(x<6), P(x \geq 6), P(3 < x \leq 6)$.

b. Find the mean and standard deviation of Binomial distribution. **07** (2 :5 : 1.2.1)

c. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks normally distributed. **07** (2 :5 : 1.2.1)

Given $P(1.2263) = 0.39$ and $P(1.4757) = 0.43$.

(OR)

10 **a.** The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents in a year (ii) more than 3 accidents in a year. **06** (2 :5 : 1.2.1)

b. Suppose X and Y are independent random variables with the following respective distribution, find the joint distribution of X and Y. also verify that $COV(X,Y) = 0$. **07** (2 :5 : 1.2.1)

x_i	1	2
$f(x_i)$	0.7	0.3

y_j	-2	5	8
$g(y_j)$	0.3	0.5	0.2

c. **07** (2 :5 : 1.2.1)

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Determine the marginal distribution of X and Y. Also compute (i) Expectations of X and Y (ii) S.D of X and Y (iii) Covariance of X and Y (iv) Correlation of X and Y. Further verify that X and Y are dependent random variables.

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