**2022 SCHEME**

*Basavarajeswari Group of Institutions*

# BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

**USN** | | | | | | | | | | |  **Course Code** | 2 | 2 | M | C | S | 3 | 1 |

Third Semester B.E. Degree Examinations, January 2025

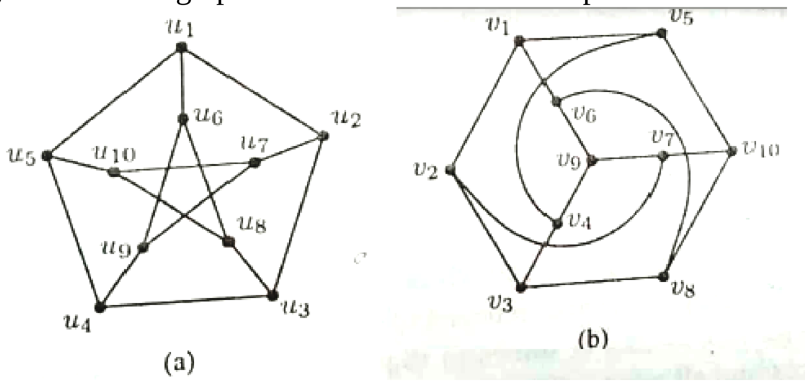
## GRAPH THEORY & DISCRETE MATHEMATICAL STRUCTURES

**(Computer Science & Engineering)**

**Duration: 3 hrs** **Max. Marks: 100**

***Note:*** *1. Answer any* **FIVE** *full questions choosing ONE Full Question from each Module*
*2. Formula Handbook is permitted*
*3. Missing data, if any, may be suitably assumed*

| Q. No | | Question | Marks | (RBTL:CO:PI) |
|---|---|---|---|---|
| | | **Module-1** | | |
| **1.** | **a.** | If $G = G(V,E)$ is a simple graph, prove that $2\|E\| \le \|V\|^2 - \|V\|$. | **06** | (2 :1 : 1.2.1) |
| | **b.** | For a graph with $n$ vertices and $m$ edges, if $\delta$ is the minimum and $\Delta$ is the maximum of the degrees of vertices, show that $\delta \le \frac{2m}{n} \le \Delta$. | **07** | (2 :1 : 1.2.1) |
| | **c.** | Verify that the two graphs shown below are isomorphic. | **07** | (2 :1 : 1.2.1) |




**(OR)**

| Q. No | | Question | Marks | (RBTL:CO:PI) |
|---|---|---|---|---|
| **2.** | **a.** | Show that the complete bipartite graph $K_{3,3}$ is a non-planar graph. | **06** | (2 :1 : 1.2.1) |
| | **b.** | Draw the following<br>(i) A graph which has both an Euler circuit and Hamilton cycle.<br>(ii) A graph which has an Euler circuit but no Hamilton cycle.<br>(iii) A graph which has a Hamilton cycle but no Euler circuit.<br>(iv) A graph which has neither a Hamilton cycle nor an Euler circuit. | **07** | (2 :1 : 1.2.1) |
| | **c.** | How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles. | **07** | (2 :1 : 1.2.1) |
| | | **Module-2** | | |
| **3.** | **a.** | Prove that, a graph G is said to be a **tree** if it is connected and has no cycles. | **06** | (2 :2 : 1.2.1) |
| | **b.** | Prove that, any connected graph with n vertices and n-1 edges is a tree. | **07** | (2 :2 : 1.2.1) |
| | **c.** | Suppose that a tree T has $N_1$ vertices of degree 2, $N_2$ vertices of degree 2, $N_3$ vertices of degree 3,…. , $N_k$ vertices of degree k.<br>Prove that $N_1 = 2 + N_3 + 2N_4 + 3N_5 + .... + (k-2) N_k$. | **07** | (2 :2 : 1.2.1) |

**(OR)**

**Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)**

**4.** **a.** A code for {a, b, c, d, e} is given by a: 00, b: 01, c: 101, d: x10, e: yz1. Where $x, y, z \in \{0, 1\}$. Determine $x, y$ *and* $z$ so that the given code is a prefix code. **06** (2 :2 : 1.2.1)

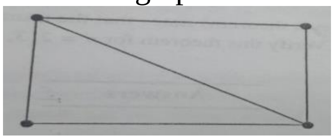
**b.** Obtain an optimal prefix code for the message MISSION SUCCESSFUL. Indicate the code for the message. **07** (2 :2 : 1.2.1)

**c.** Construct an optimal prefix code for the symbols with given weights in the following table **07** (2 :2 : 1.2.1)

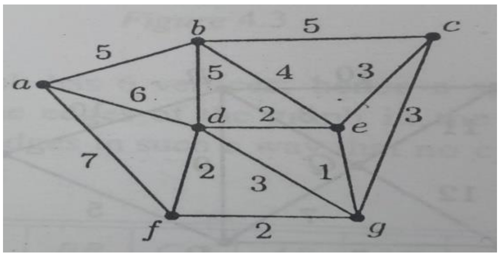
| Symbols: | a | b | c | d | e | f | g |
|---|---|---|---|---|---|---|---|
| Weight: | 10 | 30 | 5 | 15 | 20 | 15 | 5 |

## Module-3

**5.** **a.** Find all the spanning trees of the graph shown below: **06** (2 :3 : 1.2.1)



**b.** Using Kruskal's algorithm, find a minimal spanning tree for the weighted graph shown below. **07** (2 :3 : 1.2.1)



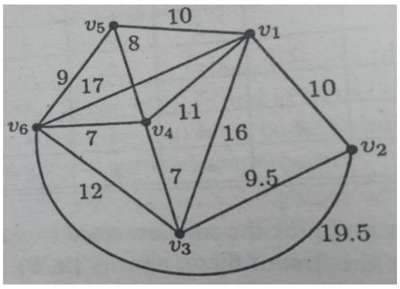

**c.** Eight cities A, B, C, D, E, F, G, H are required to be connected by a new railway network. The possible tracks and the cost of involved to lay them (in crores of rupees) are summarized in the following table: **07** (2 :3 : 1.2.1)

| Track between | Cost | Track between | Cost |
|---|---|---|---|
| A and B | 155 | D and F | 100 |
| A and D | 145 | E and F | 150 |
| A and G | 120 | F and G | 140 |
| B and C | 145 | F and H | 150 |
| C and D | 150 | G and H | 160 |
| C and E | 95 | | |

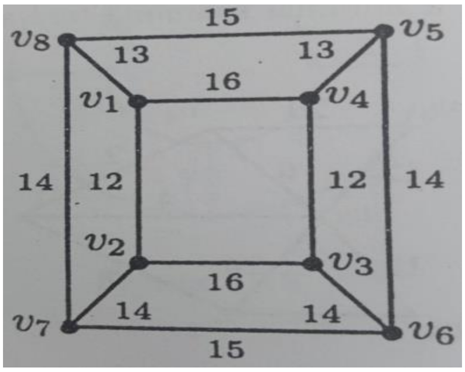
Determine a railway network of minimal cost that connects all these cities.

**(OR)**

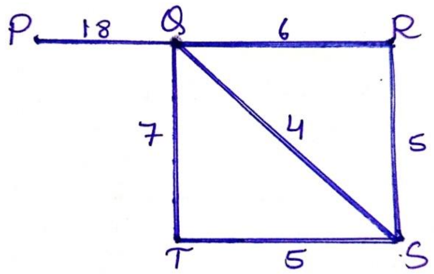
**6.** **a.** Using Prim's algorithm, find a minimal spanning tree for the weighted graph shown below. **06** (2 :3 : 1.2.1)



| | | | | |
|---|---|---|---|---|
| | **b.** | Find by Kruskal's algorithm a minimal spanning tree for the following weighted graph: | **07** | (2 :3 : 1.2.1) |




| | | | | |
|---|---|---|---|---|
| | **c.** | Find the maximum flow possible between the vertices P and S in the network shown in below figure | **07** | (2 :3 : 1.2.1) |




**(OR)**

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|---|---|---|---|---|
| **7.** | **a.** | Let $A = \{1, 2, 3, 4\}$ and let $R$ be the relation on $A$ defined by $xRy$ if and only if "$x$ divides $y$", written $x \mid y$.<br>(i) Write down $R$ as a set of ordered pairs.<br>(ii) Draw the digraph of $R$.<br>(iii) Determine the in-degrees and out-degrees of the vertices in the digraph. | **06** | (2 :4 : 1.2.1) |
| | **b.** | Let $A=\{a,b,c,d\}$ and $R$ be a relation on $A$ that has the matrix<br>$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$<br>Construct the digraph of $R$ and list the in-degrees and out-degrees of all vertices. | **07** | (2 :4 : 1.2.1) |
| | **c.** | Draw the Hasse diagram representing the positive divisors of 36. | **07** | (2 :4 : 1.2.1) |

**(OR)**

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| **8.** | **a.** | Consider the functions $f$ and g defined by $f(x) = x^3$ and $g(x) = x^2 + 1, \forall x \in R$. Find $gof$, $fog$, $f^2$ and $g^2$. | **006** | (2 :4 : 1.2.1) |
| | **b.** | Let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be a permutation of the set $A=\{1,2,3,4,5,6\}$ (i) Write $p$ as a product of disjoint cycles.<br>(ii) Compute $p^{-1}$ (iii) Compute $p^2$ and $p^3$. | **07** | (2 :4 : 1.2.1) |

**Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)**

| | | | | |
|---|---|---|---|---|
| | **c.** | Prove that if 30 dictionaries in a library contain a total of 61,327 pages, then least one of the dictionaries must have atleast 2045 pages. | **07** | (2 :4 : 1.2.1) |

**Module-5**

| | | | | |
|---|---|---|---|---|
| **9.** | **a.** | The number of virus affected files in a system is 1000 (to start with) and this increases 250 % every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. | **06** | (2 :5 : 1.2.1) |
| | **b.** | Solve the recurrence relation $2a_n - 3a_{n-1} = 0$, for $n \geq 1$, given that $a_4 = 81$. | **07** | (2 :5 : 1.2.1) |
| | **c.** | Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$, for $n \geq 2$, given that $a_0 = 5, a_1 = 12$. | **07** | (2 :5 : 1.2.1) |

**(OR)**

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|---|---|---|---|---|
| **10** | **a.** | Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given $F_0 = 0, F_1 = 1$. | **06** | (2 :5 : 1.2.1) |
| | **b.** | Solve the recurrence relation $a_{n+2} + 4a_{n+1} + 4a_n = 7$, $n \geq 0$. given that $a_0 = 1$, $a_1 = 2$. | **07** | (2 :5 : 1.2.1) |
| | **c.** | Solve the recurrence relation $a_{n+2} - 4a_{n+1} + 3a_n = -200, n \geq 0$ and $a_0 = 3000, a_1 = 3300$. | **07** | (2 :5 : 1.2.1) |

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**Note: (RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI– Performance Indicator)**