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Course Code

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First Semester B.E. Degree Examinations, February 2025

MATHEMATICS FOR MECHANICAL ENGINEERING STREAM-I

Duration: 3 hrs

Max. Marks: 100

- Note:**
1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. Missing data, if any, may be suitably assumed
 3. Use of Mathematics Formula Handbook is permitted.

<u>Q. No</u>	<u>Question</u>	<u>Marks</u>	<u>(RBTL:CO:PI)</u>
Module-1			
1. a.	Find the rank of the following matrix by row echelon form.	06	(2 : 1 : 1.2.1)
	$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$		
b.	Investigate the values of λ and μ such that the system of equations $x + y + z = 6; x + 2y + 3z = 10; x + 2y + \lambda z = \mu$ may have (i) Unique solution (ii) Infinite solution (iii) No solution.	07	(3 : 1 : 1.2.1)
c.	Solve the system of equations by Gauss elimination method $2x + y + 4z = 12; 4x + 11y - z = 33; 8x - 3y + 2z = 20$	07	(3 : 1 : 1.2.1)
(OR)			
2. a.	Solve the system of equations by Gauss-Jordan elimination method $x + y + z = 8; -x - y + 2z = -4; 3x + 5y - 7z = -14$	06	(3 : 1 : 1.2.1)
b.	Solve the system of equations using Gauss Seidel method $20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25$	07	(3 : 1 : 1.2.1)
c.	Find the dominant Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh's power method taking the initial Eigen vector as $[1 \ 1 \ 1]^T$.	07	(2 : 1 : 1.2.1)
Module-2			
3. a.	Prove with usual notations, $\tan \phi = r \frac{d\theta}{dr}$	06	(3 : 2 : 1.2.1)
b.	Find the angle between the pair of curves $r = \sin \theta + \cos \theta$ and $r = 2 \sin \theta$	07	(2 : 2 : 1.2.1)
c.	Find the pedal equation of the curve $r(1 - \cos \theta) = 2a$.	07	(2 : 2 : 1.2.1)

(OR)

4. a. For the curve $y = \frac{ax}{(a+x)}$, show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$. 06 (2 : 2 : 1.2.1)
- b. Prove with usual notation $\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$ 07 (3 : 2 : 1.2.1)
- c. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$ 07 (2 : 2 : 1.2.1)

Module-3

5. a. Expand $\log(\sec x)$ upto the term containing x^6 using Maclaurin's series. 06 (2 : 3 : 1.2.1)
- b. If $u = \log\left(\frac{x^2 + y^2}{x + y}\right)$ verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. 07 (3 : 3 : 1.2.1)
- c. If $u = \log\sqrt{x^2 + y^2 + z^2}$ show that 07 (3 : 3 : 1.2.1)
- $$(x^2 + y^2 + z^2) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 1.$$

(OR)

6. a. If $x = r \sin \theta \cos \phi$, $y = r \sin \theta \sin \phi$, $z = r \cos \theta$, show that 06 (3 : 3 : 1.2.1)
- $$\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin \theta.$$
- b. Examine the function $\sin x + \sin y + \sin(x + y)$ for extreme values. 07 (3 : 3 : 1.2.1)
- c. Write a Python Program to find partial derivatives of functions 07 (3 : 3 : 1.2.1)
- $u = e^x [x \cos(y) - y \sin(y)]$ then $U_{xx} + U_{yy} = 0$.

Module-4

7. a. Solve $r \sin \theta - \cos \theta \left(\frac{dr}{d\theta} \right) = r^2$ 06 (3 : 4 : 1.2.1)
- b. Solve $(x^2 - 4xy - 2y^2) dx + (y^2 - 4xy - 2x^2) dy = 0$. 07 (3 : 4 : 1.2.1)
- c. Find the orthogonal trajectories of the family $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$. where 07 (3 : 4 : 1.2.1)
- λ is a parameter

(OR)

8. a. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L \frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t 06 (2 : 4 : 1.2.1)
- b. Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$ 07 (3 : 4 : 1.2.1)
- c. Show that the equation $xp^2 + px - py + 1 - y = 0$ is Clairaut's equation. Hence obtain the general and singular solution 07 (2 : 4 : 1.2.1)

Module-5

9. a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dz dy dx$ 06 (3 :5 : 1.2.1)
- b. Change the order of the integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy dx$. 07 (3 :5 : 1.2.1)
- c. Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0, x+y+z=1$ 07 (2 :5 : 1.2.1)
- (OR)
10. a. Prove that $\Gamma(1/2) = \sqrt{\pi}$ 06 (3 :5 : 1.2.1)
- b. Evaluate $\int_0^{\pi/2} \sqrt{\tan \theta} d\theta$ by expressing in terms of gamma functions 07 (3 :5 : 1.2.1)
- c. Prove that $\beta(m, n) = \frac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$. 07 (3 :5 : 1.2.1)

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