BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | 2 | 2 | M | A | T | M | 1 | 1 |

First Semester B.E. Degree Summer Semester Examinations, September/October 2025

MATHEMATICS-I FOR MECHANICAL ENGINEERING STREAM

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Missing data, if any, may be suitably assumed
3. Use of Mathematics Formula Handbook is permitted.

Q. No		*Question*	*Marks*	*(RBTL:CO:PI)*
		Module-1		
1.	**a.**	Find the rank of the following matrix by row echelon form $A=\begin{bmatrix}2 & 3 & -1 & -1\\ 1 & -1 & -2 & -4\\ 3 & 1 & 3 & -2\\ 6 & 3 & 0 & 7\end{bmatrix}$	**06**	**(1 :1 : 1.2.1)**
	b.	Investigate the values of λ and μ such that the system of equations x+y+z=6; x+2y+3z=10, x+2y+ $\lambda z = \mu$ may have (i) Unique solution (ii) Infinite solution (iii) No solution.	**07**	**(2 :1 : 1.2.1)**
	c.	Solve the following system of equations by Gauss elimination method 2x+y+4z=12, 4x+11y-z=33, 8x-3y+2z=20.	**07**	**(3 :1 : 1.2.1)**
		(OR)		
2.	**a.**	Solve the following system of equations by Gauss-Jordon elimination method $2x+3y-z=5$; $4x+4y-3z=3$; $2x-3y+2z=2$.	**06**	**(3 :1 : 1.2.1)**
	b.	Solve the following system of equations using Gauss Seidel method $20x+y-2z=17$; $3x+20y-z=-18$; $2x+3y+20z=25$	**07**	**(3 :1 : 1.2.1)**
	c.	Find the dominant Eigen value and the corresponding Eigen vector of the matrix A=$\begin{bmatrix}2 & 0 & 1\\ 0 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$ by Rayleigh's power method taking the initial Eigen vector as $\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$.	**07**	**(3 :1 : 1.2.1)**
		Module-2		
3.	**a.**	Prove that for the curve $r=f(\theta)$ $\frac{1}{p^2}=u^2+\left(\frac{du}{d\theta}\right)^2$ *where* $u=\frac{1}{r}$.	**06**	**(2: 2 : 1.2.1)**
	b.	Find the angle between the pairs of curves $r=a(1+\cos\theta)$ and $r=b(1-\cos\theta)$	**07**	**(3 :2 : 1.2.1)**
	c.	Find the pedal equation of the curve $r^n=a^n\cos\theta$	**07**	**(3 :2 : 1.2.1)**
		(OR)		

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

4. **a.** Prove with usual notation: $\rho = \dfrac{\left(1+y_1^2\right)^{3/2}}{y_2}$ **06** **(2 :2 : 1.2.1)**

b. Find the radius of curvature of $a^2 y = x^3 - a^3$ where the curve cuts the x-axis. **07** **(3 :2 : 1.2.1)**

c. Find the radius of curvature of the curve $r^n = a^n \cos n\theta$. **07** **(3 :2 : 1.2.1)**

Module-3

5. **a.** Obtain the Maclaurin's series expansion of $e^{\sin x}$ upto the term containing x^4. **06** **(2 :3 : 1.2.1)**

b. If $z = f\left(x+ct\right)+g\left(x-ct\right)$, prove that $\dfrac{\partial^2 z}{\partial t^2} = c^2 \dfrac{\partial^2 z}{\partial x^2}$. **07** **(2 :3 : 1.2.1)**

c. If $u = \log\left(x^3+y^3+z^3-3xyz\right)$, then prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z} = \dfrac{3}{x+y+z}$. **07** **(2 :3 : 1.2.1)**

(OR)

6. **a.** If $u = f\left(x-y, y-z, z-x\right)$, show that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z} = 0$ **06** **(2 :3 : 1.2.1)**

b. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$ then find $\dfrac{\partial\left(u,v,w\right)}{\partial\left(x,y,z\right)}$. **07** **(3 :3 : 1.2.1)**

c. Examine the function $z\left(x,y\right) = x^4+y^4-2\left(x-y\right)^2$ for extreme values. **07** **(2 :3 : 1.2.1)**

Module-4

7. **a.** Solve: $6y^2 dx - x(x^3+2y)dy = 0$ **06** **(3 :4 : 1.2.1)**

b. Solve: $ye^{xy}dx + (xe^{xy}+2y)dy = 0.$ **07** **(3 :4 : 1.2.1)**

c. Find the orthogonal trajectories of the family $x^{2/3}+y^{2/3} = a^{2/3.}$ **07** **(1 :4 : 1.2.1)**

(OR)

8. **a.** A series circuit with resistance 'R' inductance 'L' and electromotive force 'E' is governed by the differential equation $L\dfrac{di}{dt}+Ri = E$ Where R and L are constants and initially current i = 0. Find the current at any time't'. **06** **(1 :4 : 1.2.1)**

b. Solve: $xy\left(\dfrac{dy}{dx}\right)^2 - (x^2+y^2)\dfrac{dy}{dx} + xy = 0$ **07** **(3 :4 : 1.2.1)**

c. Modify the following equation into Clairut's form. Hence obtain the associated general and singular solutions. $xp^2 + px - py + 1 - y = 0$ **07** **(2 :4 : 1.2.1)**

Module-5

9. **a.** Evaluate : $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz.$ **06** **(3 :5 : 1.2.1)**

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

b. Change the order of the integration and hence evaluate $\int_{0}^{1}\int_{\sqrt{y}}^{2-y} xy\ dxdy$. **07** **(3 :5 : 1.2.1)**

c. Write a Python Program to find $\int_{0}^{1}\int_{x}^{\sqrt{x}} \left(x^2 + y^2\right) dy\ dx$ **07** **(1 :5 : 1.2.1)**

(OR)

10. a. Prove that: $\Gamma\left(1/2\right) = \sqrt{\pi}$ **06** **(1 :5 : 1.2.1)**

b. Prove that $\beta(m,n) = \dfrac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$. **07** **(1 :5 : 1.2.1)**

c. Express the following integrals in terms of beta functions and hence evaluate $\int_{0}^{\infty}\dfrac{dx}{1+x^4}$. **07** **(1 :5 : 1.2.1)**

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