Basavarajeswari Group of Institutions **2024 SCHEME**

BALLARI INSTITUTE OF TECHNOLOGY & MANAGEMENT

(Autonomous Institute under Visvesvaraya Technological University, Belagavi)

USN | | | | | | | | | | | **Course Code** | M | M | C | 1 | 0 | 2 |

First Semester MCA Degree Examinations, April 2025

DISCRETE MATHEMATICS, GRAPH THEORY, PROBABILITY AND STATISTICS

Duration: 3 hrs **Max. Marks: 100**

Note: ***1. Answer any* FIVE *full questions, choosing* ONE *full question from each module.***
2. Use of Mathematics Formula Handbook is permitted.
3. Missing data, if any, may be suitably assumed

Q. No		Question	Marks	(RBTL:CO: PI)
		MODULE – 1		
1.	**a.**	Let $A=\{1,2,3,4\}$ and let R be the relation on A defined by xRy iff y=2x. (i) Write down R as set of ordered pairs. (ii) Draw the digraph of R. (iii)Determine the in-degrees and out-degrees of the vertices in the digraph	**06**	**(2 :1 : 1.2.1)**
	b.	Let $A=\{a,b,c,d\}$ and R be a relation on A that has the matrix $M_R=\begin{bmatrix}1&0&0&0\\0&1&0&0\\1&1&1&0\\0&1&0&1\end{bmatrix}$ Construct the digraph of R and list the in-degrees and out-degrees of all vertices.	**07**	**(2 :1 : 1.2.1)**
	c.	Let $A=\{1,2,3,4,6,12\}$. On A, define the relation R by aRb if and only if a divides b. Prove that R is a partial order on A. Draw the Hasse diagram for this relation.	**07**	**(1 :1: 1.2.1)**
		(OR)		
2	**a.**	Let f and g be functions from R to R defined by $f(x)=ax+b$ and $g(x)=cx+d$. What relationship must be satisfied by a,b,c,d if $gof = fog$.	**06**	**(2 :1 : 1.2.1)**
	b.	Let $A=\{1,2,3,4,5,6\}$. Compute $(4,1,3,5)o(5,6,3)$ and $(5,6,3)o(4,1,3,5)$.	**07**	**(2 :1: 1.2.1)**
	c.	Let $p=\begin{bmatrix}1&2&3&4&5&6\\4&3&5&1&2&6\end{bmatrix}$ be a permutation of the set $A=\{1,2,3,4,5,6\}$. (a) Write p as a product of disjoint cycles. (b) Compute p^{-1} (c) Compute p^2 and p^3 (d) Find the smallest positive integer k such that $p^k=I_A$.	**07**	**(1 :1: 1.2.1)**

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

MODULE – 2

			Marks	(RBTL:CO:PI)
3	a.	Find the rank of the following matrix by elementary row transformations $A=\begin{bmatrix}1&2&4&3\\2&4&6&8\\4&8&12&16\\1&2&3&4\end{bmatrix}$	06	(1 :2: 1.2.1)
	b.	Test for consistency and solve the following system of equations $x+2y+2z=1; 2x+y+z=2; 3x+2y+2z=3; y+z=0$	07	(1 :2: 1.2.1)
	c.	Solve the following system of equations by Gauss- elimination method $x-2y+3z=2; 3x-y+4z=4; 2x+y-2z=5$	07	(1 :2: 1.2.1)
		(OR)		
4	a.	Solve the following system of equations by Gauss- Jordan elimination method $2x+5y+7z=52; 2x+y-z=0; x+y+z=9$	06	(1 :2: 1.2.1)
	b.	Find the numerically largest Eigen value and the corresponding Eigen vector of the matrix $A=\begin{bmatrix}4&1&-1\\2&3&-1\\-2&1&5\end{bmatrix}$ by taking the initial approximation to the Eigen vector as $\begin{bmatrix}1 & 0.8 & -0.8\end{bmatrix}^T$ by Rayleigh's Power method. Perform 5 iterations.	07	(1 :2: 1.2.1)
	c.	Find the inverse transformation of the following linear transformation $y_1=2x_1-2x_2-x_3; y_2=-4x_1+5x_2+3x_3; y_3=x_1-x_2-x_3$.	07	(1 :2: 1.2.1)

MODULE – 3

			Marks	(RBTL:CO:PI)
5	a.	Let p : A circle is a conic. $q:\sqrt{5}$ is an irrational number. r : Exponential series is convergent. Express the following compound propositions in words. (i) $p\wedge(\sim q)$ (ii) $(\sim p)\vee q$ (iii) $p\to(q\vee r)$ (iv) $\sim p\leftrightarrow\{q\wedge(\sim r)\}$	06	(1 :3: 1.2.1)
	b.	Construct the truth tables of the following compound propositions: (i) $(p\wedge q)\to\sim r$ (ii) $q\wedge(\sim r\to p)$	07	(1 :3: 1.2.1)
	c.	Prove that, for any propositions p, q, r, the compound proposition $\{p\to(q\to r)\}\to\{(p\to q)\to(p\to r)\}$ is a tautology.	07	(1 :3: 1.2.1)
		(OR)		
6.	a.	Prove that, for any propositions p and q, the compound propositions $p\veebar q$ and $(p\vee q)\wedge(\sim p\vee\sim q)$ are logically equivalent.	06	(1 :3: 1.2.1)
	b.	Test the validity of the following argument: ***If I study, I will not fail in the examination.*** ***If I do not watch TV in the evenings, I will study.*** ***I failed in the examination.*** ——— ***Therefore, I must have watched TV in the evenings.***	07	(1 :3: 1.2.1)

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

c. Prove that the following argument is valid: **07** **(1 :3: 1.2.1)**

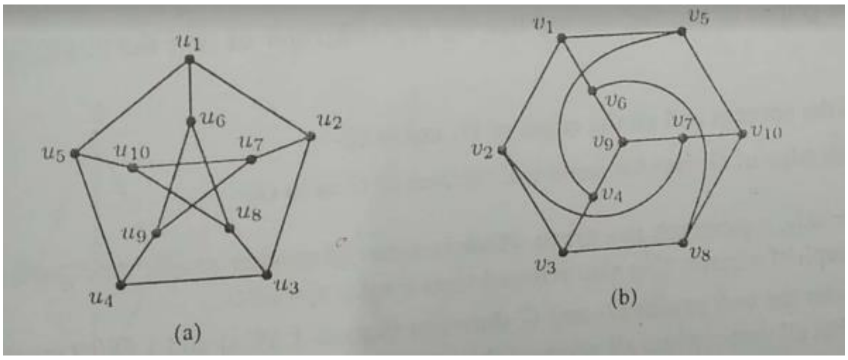
$$\begin{array}{l} \forall x, \left[p(x) \vee q(x) \right] \\ \exists x, \sim p(x) \\ \forall x, \left[\sim q(x) \vee r(x) \right] \\ \forall x, \left[s(x) \rightarrow \sim r(x) \right] \\ \hline \therefore \exists x, \sim s(x) \end{array}$$

MODULE – 4

7. **a.** If $G = G(V, E)$ is a simple graph, prove that $2|E| \leq |V|^2 - |V|$. **06** **(2 :4: 1.2.1)**

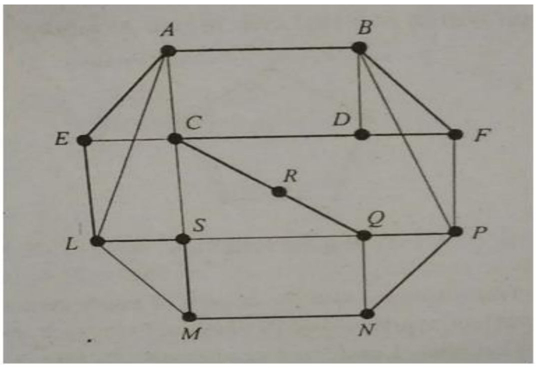
b. For a graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degrees of vertices, show that $\delta \leq \dfrac{2m}{n} \leq \Delta$. **07** **(2 :4: 1.2.1)**

c. Show that the following two graphs are isomorphic: **07** **(2 :4: 1.2.1)**



(OR)

8. **a.** Show that the graph shown in below is a Hamilton graph. **06** **(2 :4: 1.2.1)**



b. Exhibit the following **07** **(2 :4: 1.2.1)**

(i) *A graph which has both an Euler circuit and Hamilton cycle.*
(ii) A graph which has an Euler circuit but no Hamilton cycle.
(iii) A graph which has a Hamilton cycle but no Euler circuit.
(iv) A graph which has neither a Hamilton cycle nor an Euler circuit.

c. How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also, draw the graph to show these Hamilton cycles. **07** **(2 :4: 1.2.1)**

Note: (**RBTL - Revised Bloom's Taxonomy Level: CO - Course Outcome: PI - Performance Indicator)**

MODULE – 5

9. **a.** A random variable X has the following probability functions for the various values of x. **06** **(1 :5: 1.2.1)**

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k (ii) Evaluate $P(x<6), P(x\geq 6), P(3<x\leq 6)$.

b. Find the mean and standard deviation of Binomial distribution. **07** **(1 :5: 1.2.1)**

c. In a certain factory turning out razor blades there is a small probability of 1/500 for any blade to be defective. The blades are supplied in a packet of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) no defective (ii) one defective (iii) two defective blades in a consignment of 10,000 packets. **07** **(1 :5: 1.2.1)**

(OR)

10. **a.** Find the value of 'c' such that $f(x)=\begin{cases}\frac{x}{6}+c, 0\leq x\leq 3\\ 0 \quad \text{elsewhere}\end{cases}$ is a p.d.f. **06** **(1 :5: 1.2.1)**

Also find $P(1\leq x\leq 2)$.

b. If x is an exponential variate with mean 3 find **07** **(1 :5: 1.2.1)**
(i) $P(x>1)$ (ii) $P(x<3)$

c. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks normally distributed. **07** **(1 :5: 1.2.1)**

Given $P(1.2263)=0.39$ and $P(1.4757)=0.43$

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